

UNIFIED SEMICLASSICAL APPROACH TO ISOSCALAR COLLECTIVE MODES IN HEAVY NUCLEI

V. I. ABROSIMOV

Institute for Nuclear Research, 03028 Kiev, Ukraine
E-mail: abrosim@kinr.kiev.ua

A. DELLAFIORE AND F. MATERA

Istituto Nazionale di Fisica Nucleare and Dipartimento di Fisica,
Università di Firenze, via Sansone 1, 50019 Sesto F.no (Firenze), Italy
E-mail: della@fi.infn.it; matera@fi.infn.it

A semiclassical model based on the solution of the Vlasov equation for finite systems with a sharp moving surface has been used to study the isoscalar quadrupole and octupole collective modes in heavy spherical nuclei. Within this model, a unified description of both low-energy surface modes and higher-energy giant resonances has been achieved by introducing a coupling between surface vibrations and the motion of single nucleons. Analytical expressions for the collective response functions of different multipolarity can be derived by using a separable approximation for the residual interaction between nucleons. The response functions obtained in this way give a good qualitative description of the quadrupole and octupole response in heavy nuclei. Although shell effects are not explicitly included in the theory, our semiclassical response functions are very similar to the quantum ones. This happens because of the well known close relation between classical trajectories and shell structure. The role played by particular nucleon trajectories and their connection with various features of the nuclear response is displayed most clearly in the present approach, we discuss in some detail the damping of low-energy octupole vibrations and give an explicit expression showing that only nucleons moving on triangular orbits can contribute to this damping.

1. Introduction

It is well known that the isoscalar quadrupole and octupole response of nuclei displays both low- and high-energy collective modes¹. Also known is that semiclassical models have difficulties in describing both these systematic features of the isoscalar response, in particular, models based on fluid dynamics, see e.g.², can explain the giant resonances, but fail to describe the low-energy collective modes. On the other hand it is known from quan-

tum studies that the coupling between the motion of individual nucleons and surface vibrations plays an essential role in low-energy nuclear collective modes, see e.g. ^{3,4,5}. Semiclassical models of the fluid-dynamical type do not contain explicitly the single-particle degrees of freedom, so they can not describe the coupling between individual nucleons and surface motion.

In the present contribution we review a study the isoscalar collective modes in nuclei ^{6,7} made by using a semiclassical approach that includes the single-particle degrees of freedom explicitly and thus allows for an account the coupling between individual nucleons and surface motion. Our model is based on the linearized Vlasov kinetic equation for finite systems with moving surface ^{8,9}. The coupling between the motion of individual nucleons and the surface vibrations is obtained by treating the nuclear surface as a collective dynamical variable, like in the liquid drop model. Here we concentrate our attention on the isoscalar quadrupole and octupole collective modes in heavy spherical nuclei, an application of the same model to the compression dipole modes has been discussed in a previous meeting of this series ¹⁰.

2. Reminder of formalism

This Section recalls briefly the formalism of References ^{8,9} which is at the basis of the present approach.

The fluctuations of the phase-space density induced by a weak external force can be described by the linearized Vlasov equation, which is usually a differential equation in seven variables. For spherical systems this equation can be reduced to a system of two (coupled) differential equations in the radial coordinate alone ⁸. This is achieved by means of a change of variables and a partial-wave expansion:

$$\begin{aligned} \delta f(\mathbf{r}, \mathbf{p}, \omega) = & \sum_{LMN} [\delta f_{MN}^{L+}(\epsilon, \lambda, r, \omega) + \delta f_{MN}^{L-}(\epsilon, \lambda, r, \omega)] \\ & \times \left(\mathcal{D}_{MN}^L(\alpha, \beta, \gamma) \right)^* Y_{LN}\left(\frac{\pi}{2}, \frac{\pi}{2}\right). \end{aligned} \quad (1)$$

The functions $\delta f_{MN}^{L\pm}(\epsilon, \lambda, r, \omega)$ are partial-wave components of the (Fourier transformed in time) density fluctuations for particles with energy ϵ , magnitude of angular momentum λ and radial position r , the \pm sign distinguishes between particles having positive or negative components of the radial momentum p_r .

The other terms in the expansion are Wigner matrices and spherical harmonics.

In order to solve the one-dimensional linearized Vlasov equation for the $\delta f_{MN}^{L\pm}$ functions we must specify the boundary conditions satisfied by these functions. Different boundary conditions allow us to study different physical properties of the system, so the fixed-surface boundary conditions employed in ⁸ were adequate to study giant resonances, but different (moving-surface) boundary conditions ⁹ must be introduced in order to study surface modes. We assume that the external force can also induce oscillations of the system surface according to the usual liquid-drop model expression

$$R(\vartheta, \varphi, t) = R + \sum_{LM} \delta R_{LM}(t) Y_{LM}(\vartheta, \varphi) \quad (2)$$

and the boundary condition satisfied by the functions $\delta f_{MN}^{L\pm}$ at the nuclear surface is taken as

$$\delta f_{MN}^{L+}(R) - \delta f_{MN}^{L-}(R) = 2F'(\epsilon) i\omega p_r \delta R_{LM}(\omega). \quad (3)$$

This equation has been derived with the assumption that the equilibrium phase-space density is a function $F(\epsilon)$ of the particle energy alone, $F'(\epsilon)$ is its derivative. The boundary condition (3) corresponds to a mirror reflection of particles in the reference frame of the moving nuclear surface, it provides a coupling between the motion of nucleons and the surface vibrations. A self-consistency condition involving the nuclear surface tension is then used to determine the time (or frequency) dependence of the additional collective variables $\delta R_{LM}(t)$ ⁹.

Now, assuming a simplified residual interaction of separable form,

$$v(r_1, r_2) = \kappa_L r_1^L r_2^L, \quad (4)$$

the moving-surface isoscalar collective response function of a spherical nucleus, described as a system of A interacting nucleons contained in a cavity of equilibrium radius $R = 1.2A^{\frac{1}{3}}$ fm, is given by

$$\tilde{\mathcal{R}}_L(s) = \mathcal{R}_L(s) + \mathcal{S}_L(s). \quad (5)$$

Instead of the frequency ω , as independent variable we have used the more convenient dimensionless quantity $s = \omega/(v_F/R)$ (v_F is the Fermi velocity). The response function $\mathcal{R}_L(s)$, given by

$$\mathcal{R}_L(s) = \frac{\mathcal{R}_L^0(s)}{1 - \kappa_L \mathcal{R}_L^0(s)}, \quad (6)$$

describes the collective response in the fixed-surface limit. The response function $\mathcal{R}_L^0(s)$ is analogous to the quantum single-particle response func-

tion and it is given explicitly by ^{6,7},

$$\mathcal{R}_L^0(s) = \frac{9A}{8\pi} \frac{1}{\epsilon_F} \sum_{n=-\infty}^{+\infty} \sum_{N=-L}^{N=L} (C_{LN})^2 \int_0^1 dx x^2 s_{nN}(x) \frac{(Q_{nN}^{(L)}(x))^2}{s + i\varepsilon - s_{nN}(x)}, \quad (7)$$

where ϵ_F is the Fermi energy and the quantity ε is a vanishingly small parameter that determines the integration path at poles.

The functions $s_{nN}(x)$ are defined as

$$s_{nN}(x) = \frac{n\pi + N \arcsin(x)}{x}. \quad (8)$$

The variable x is related to the classical nucleon angular momentum λ . The quantities C_{LN} in Eq. (7) are classical limits of the Clebsh-Gordan coefficients coming from the angular integration. In principle the integer N takes values between $-L$ and L , however only the coefficients C_{LN} where N has the same parity as L are nonvanishing. The coefficients $Q_{nN}^{(L)}(x)$ appearing in the numerator of Eq. (7) have been defined in Ref. ⁸, they are essentially the classical limit of the radial matrix elements of the multipole operator r^L and can be evaluated analytically for $L = 2, 3$.

The function $\mathcal{S}_L(s)$ in Eq. (5) gives the moving-surface contribution to the response. With the simple interaction (4) this function can be evaluated explicitly as ^{6,7}

$$\mathcal{S}_L(s) = - \frac{R^6}{1 - \kappa_L \mathcal{R}_L^0(s)} \frac{[\chi_L^0(s) + \kappa_L \varrho_0 R^L \mathcal{R}_L^0(s)]^2}{[C_L - \chi_L(s)][1 - \kappa_L \mathcal{R}_L^0(s)] + \kappa_L R^6 [\chi_L^0(s) + \varrho_0 R^L]^2}, \quad (9)$$

with $C_L = \sigma R^2(L-1)(L+2) + (C_L)_{coul}$, $\sigma \approx 1 \text{ MeV fm}^{-2}$ is the surface tension parameter obtained from the mass formula, $(C_L)_{coul}$ gives the Coulomb contribution to the restoring force and $\varrho_0 = A/\frac{4\pi}{3}R^3$ is the equilibrium density. The functions $\chi_L^0(s)$ and $\chi_L(s)$ are given by ¹¹

$$\chi_L^0(s) = \frac{9A}{4\pi} \frac{1}{R^3} \sum_{n=-\infty}^{+\infty} \sum_{N=-L}^{N=L} (C_{LN})^2 \int_0^1 dx x^2 s_{nN}(x) \frac{(-)^n Q_{nN}^{(L)}(x)}{s + i\varepsilon - s_{nN}(x)}, \quad (10)$$

and

$$\chi_L(s) = - \frac{9A}{2\pi} \epsilon_F (s + i\varepsilon) \sum_{nN} (C_{LN})^2 \int_0^1 dx x^2 \frac{1}{s + i\varepsilon - s_{nN}(x)}, \quad (11)$$

their structure is similar to that of the zero-order propagator (7).

We refer to the papers ^{6,7} for further details on the formalism and discuss here only the main points.

Equation (9) is the main result in the present context. Together with Eqs.(5) and (6), this equation gives a unified expression of the isoscalar response function, including both the low- and high-energy collective excitations. By comparing the fixed- and moving-surface response functions, we can appreciate the effects due to the coupling between the motion of individual nucleons and the surface vibrations.

3. Fixed- vs. moving-surface strength distributions

The strength function $S_L(E)$ associated with the response function (5) is defined as ($E = \hbar\omega$)

$$S_L(E) = -\frac{1}{\pi} \text{Im} \tilde{\mathcal{R}}_L(E). \quad (12)$$

We discuss here the isoscalar quadrupole and octupole strength distributions. The strength κ_L of the residual interaction (4) can be estimated in a self-consistent way, giving (¹², p. 557),

$$\kappa_{BM} = -\frac{4\pi}{3} \frac{m\omega_0^2}{AR^{2L-2}}, \quad (13)$$

with the parameter ω_0 given by $\omega_0 \approx 41A^{-\frac{1}{3}}\text{MeV}$. Since this estimate is based on a harmonic oscillator mean field and we are assuming a square-well potential instead, we expect some differences. Hence we determine the parameter κ_L phenomenologically, by requiring that the peak of the high-energy resonance agrees with the experimental value of the giant multipole resonance energy. This requirement implies $\kappa_L \approx 2\kappa_{BM}$ for $L = 2, 3$.

In Fig.1 we display the quadrupole strength function ($L=2$ in Eq. (12)) obtained for $A = 208$ nucleons by using different approximations. The dotted curve is obtained from the zero-order response function (7), it is similar to the quantum response evaluated in the Hartree-Fock approximation. The dashed curve is obtained from the collective fixed-surface response function (6). Comparison with the dotted curve clearly shows the effects of collectivity. The collective fixed-surface response has one giant quadrupole peak. Our result for this peak is very similar to that of the recent random-phase approximation (RPA) calculations of ¹³ (cf. Fig.5 of ¹³). However, contrary to the RPA calculations, there is no signal of a low-energy peak in the fixed-surface response function. The solid curve instead shows the moving-surface response given by Eqs. (5) and (9). Now a broad bump appears in the low-energy part of the response and a narrower peak is still present at the giant resonance energy. Of course the details of the

low-energy excitations are determined by quantum effects, nonetheless the present semiclassical approach does reproduce the average behaviour of this systematic feature of the quadrupole response.

We finally notice that the width of the giant quadrupole resonance is underestimated by our approach, this is a well known limit of all mean-field calculations that include only Landau damping. A more realistic estimate of the giant-resonance width would require including a collision term into our kinetic equation.

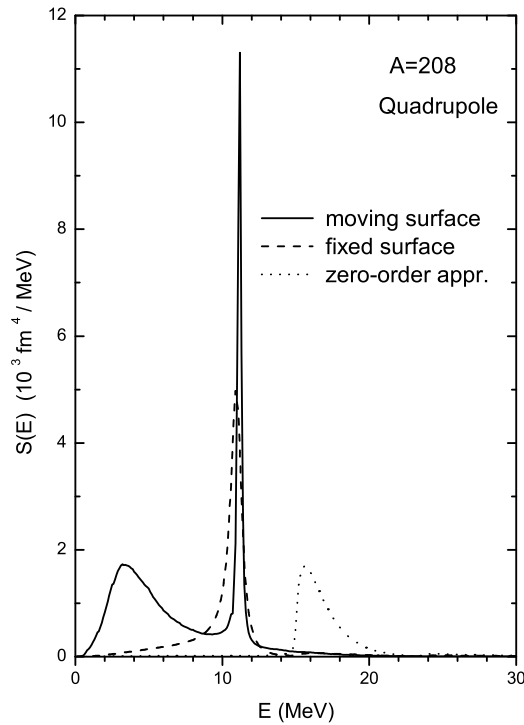


Figure 1. Quadrupole strength function for a hypothetical nucleus of $A = 208$ nucleons. The dotted curve shows the zero-order approximation, the dashed curve instead shows the collective response evaluated in the fixed-surface approximation. The full curve gives the moving-surface response.

In Fig.2 we show the octupole strength function ($L = 3$ in Eq. (12)).

The zero-order octupole strength function (dotted curve) is concentrated in two regions around 8 and 24 MeV. In this respect our semiclassical response is strikingly similar to the quantum response, which is concentrated in the $1\hbar\omega$ and $3\hbar\omega$ regions. This concentration of strength is quite remarkable because our equilibrium phase-space density, which is taken to be of the Thomas-Fermi type, does not include any shell effect, however we still obtain a strength distribution that is very similar to the one usually interpreted in terms of transitions between different shells.

We can clearly see that the collective fixed-surface response given by Eq. (6) (dashed curve) has two sharp peaks around 20 MeV and 6-7 MeV. The experimentally observed ¹ concentration of isoscalar octupole strength in the two regions usually denoted by HEOR (high energy octupole resonance) and LEOR (low energy octupole resonance) is qualitatively reproduced, however the considerable strength experimentally observed at lower energy (low-lying collective states) is absent from the fixed-surface response function. The most relevant change induced by the moving surface (solid curve in Fig.2) is the large double hump appearing at low energy. This feature is in qualitative agreement both with experiment ¹ and with the result of RPA-type calculations (see e.g. ¹⁴). We interpret this low-energy double hump as a superposition of surface vibration and LEOR.

The moving-surface octupole response of Fig. 2 displays also a novel resonance-like structure between the LEOR and the HEOR (at about 13 MeV for a system of $A = 208$ nucleons).

We also find ¹⁵ that the parameters $\delta R_{3M}(t)$, describing the octupole surface vibrations in Eq. (2), approximately satisfy an equation of motion of the damped oscillator kind:

$$D_3\delta\ddot{R}_{3M}(t) + \gamma_3\delta\dot{R}_{3M}(t) + C_3\delta R_{3M}(t) = 0. \quad (14)$$

The friction coefficient γ_L can be evaluated analytically in the low-frequency limit, giving (for a generic L)

$$\gamma_L = \gamma_{wf} 2 \frac{(4\pi)^2}{2L+1} \sum_{N=1}^L \frac{1}{N} |Y_{LN}(\frac{\pi}{2}, \frac{\pi}{2})|^2 \sum_{n=1}^{+\infty} \cos \alpha_{nN} \sin^3 \alpha_{nN} \Theta(\frac{\pi}{2} - \alpha_{nN}), \quad (15)$$

with $\gamma_{wf} = \frac{3}{4}\varrho_0 p_F R^4$ and $\alpha_{nN} = \frac{n}{N}\pi$. The angles α_{nN} are related to the nucleon trajectories. In the octupole case the coefficient $\gamma_{L=3}$ gets a contribution only from the term with $n = 1$ and $N = 3$, thus we see that only nucleons moving along closed triangular trajectories can contribute to the damping of surface octupole vibrations.

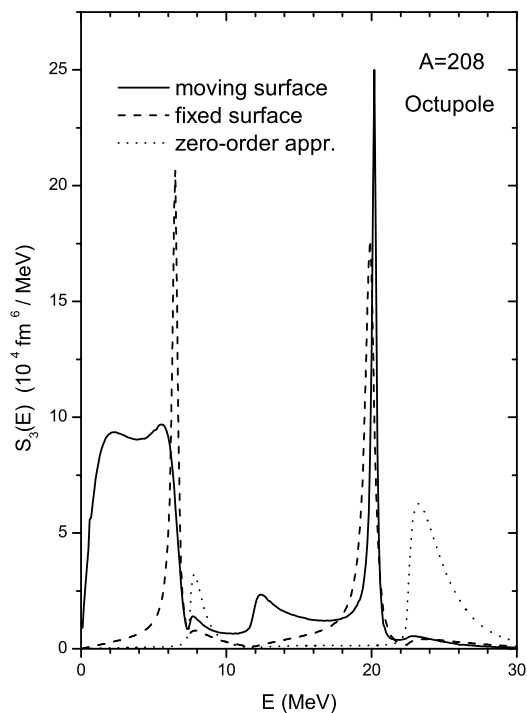


Figure 2. The same as in Fig.1 for the octupole strength function.

4. Conclusions

A unified description of the low- and high-energy isoscalar collective quadrupole and octupole response has been achieved by using appropriate boundary conditions for the fluctuations of the phase-space density described by the linearized Vlasov equation. The response functions obtained in this way give a good qualitative description of all the main features of the isoscalar response in heavy nuclei, i. e. low-lying quadrupole and octupole collective modes, plus quadrupole and octupole giant resonances. In our model the low-energy modes are surface oscillations and the coupling between single-particle motion and surface vibrations is described by simple analytical expressions.

References

1. A. van der Woude, *Progr. Part. Nucl. Phys.* **18**, 217 (1987).
2. G. Holzwarth and G. Eckart, *Nucl. Phys.* **A325**, 1 (1979).
3. P. F. Bortignon and R. A. Broglia, *Nucl. Phys.* **A371**, 405 (1981).
4. G. F. Bertsch and R. A. Broglia, *Oscillations in finite quantum systems, ch.6* (Cambridge University Press, Cambridge, UK, 1994).
5. D. Lacroix, S. Ayik and Ph. Chomaz, *Phys. Rev.* **C63**, 064305 (2001).
6. V. I. Abrosimov, A. Dellafiore and F. Matera, *Nucl. Phys.* **A717**, 44 (2003).
7. V. I. Abrosimov, O. I. Davidovskaya, A. Dellafiore and F. Matera, *Nucl. Phys.* **A727**, 220 (2003).
8. D. M. Brink, A. Dellafiore and M. Di Toro, *Nucl. Phys.* **A456**, 205 (1986).
9. V. Abrosimov, M. Di Toro and V. Strutinsky, *Nucl. Phys.* **A562**, 41 (1993).
10. V. I. Abrosimov, A. Dellafiore and F. Matera, in *Proc. of the 7th Intern. Spring Seminar on Nucl. Physics*, edited by A. Covello (World Scientific, Singapore, 2002), p.481.
11. V. I. Abrosimov, A. Dellafiore and F. Matera, *Nucl. Phys.* **A697**, 748 (2002).
12. A. Bohr and B. M. Mottelson, *Nuclear Structure, Vol. 2* (W.A. Benjamin, Inc.: Reading, Massachusetts, 1975).
13. I. Hamamoto, H. Sagawa and X. Z. Zhang, *Nucl. Phys.* **A648**, 203 (1999).
14. K. F. Liu, H. Luo, Z. Ma, Q. Shen and S. A. Moszkowski, *Nucl. Phys.* **A534**, 1 (1991).
15. V. Abrosimov, A. Dellafiore and F. Matera, *Nucl. Phys.* **A653**, 115 (1999).